Different ways of approximating the value of the constant π

Ziv Moran and Elai Corem
Supervisor: Shlomo Rozenfeld

Introduction

π is a mathematical constant widely used in all areas of mathematics and physics. Since π is an irrational number it cannot be expressed as a fraction and its decimal representation has no ending digits. Due to this, π is only able to be approximated through various techniques which go back to ancient civilizations such as the Egyptians, Babylonians and ancient Greek.

In our project, we analyzed six of the different methods that have been discovered to approximate π and compared their efficiency. We have also studied the distribution of the digits of π in order to find out whether the appearance frequency of different digits of π gets lower as we increase the number of digits after the decimal point.

The methods

From the numerous methods of approximating the value of π we have decided to study the following six:

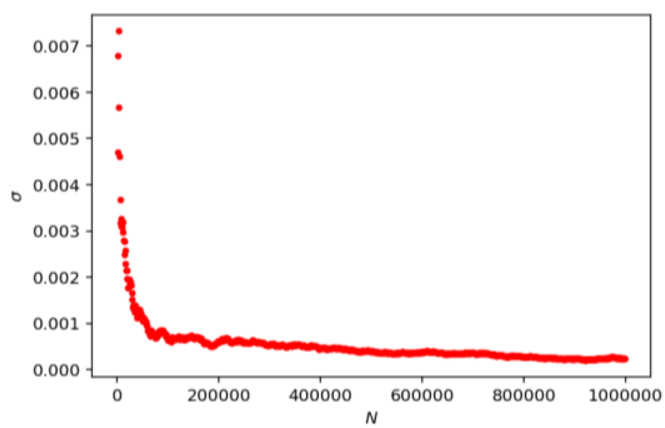
(1) The Monte Carlo method: Generating random points inside a square and calculating the ratio between the number of points generated and the number points that are inside the circle bounded by the square.
(2) Markov's sampling: Starting at a point inside a circle bounded in a square and "stepping" in random directions, creating a point at each step and calculating the ratio between the total number of points and the number of points inside the bounded circle.
(3) Colliding Blocks: Simulates a pair of blocks that move endlessly on a frictionless plane with a wall on one side and calculates the amount of collisions between the two blocks and the collisions between the blocks and the wall.
(4) Buffon's needle: "Dropping" needles on a plane divided into sections and calculating the probability that a needle would land on one of the lines separating the sections.
(5) Regular polygon: The oldest of the bunch, creates a regular polygon with a very large number of sides and calculates its perimeter.
(6) Infinite series: Calculating the infinite series sum $\sum_{n=0}^{\infty} \frac{1}{n^2}$ which is also achieved by declaring s=2 in the Zeta function (ζ).

Conclusion and result

The comparison of time needed to compute approximations of π with different methods.

Regular polygon	3.19th millionth of a second
Colliding blocks	193 millionth of a second
Infinite series	287 millionth of a second
The Monte Carlo Method	2.56 seconds
Buffon's needle	6.97 seconds
Markov's sampling	9.84 seconds

A graph representing the standard deviation in relation to the number of digits of PI after the decimal point



From these results we can derive two main conclusions.

1. Using the Regular polygon method of approximating the value of π is the fastest way to compute an approximation of π (out of all the methods we have researched in this project).
2. As we increase the number of π digits after the decimal point the standard deviation of digits approaches zero, meaning there are no digits who appear substantially more frequently than others.

As well as the conclusion that π is more than just any normal mathematical constant. It appears in the most unexpected of places. It is a most fascinating mystery.